

# A Robust Quantum Random Number Generator Based on Bosonic Stimulation

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We propose a method to realize a robust quantum random number generator based on bosonic stimulation. A particular implementation that employs weak coherent pulses and conventional avalanche photo-diode detectors (APDs) is discussed.

## I. INTRODUCTION

Random numbers are crucial for various tasks, among them generating cryptographic secret keys, authentication, Monte-Carlo simulations, digital signatures, statistical sampling, etc. Random number generators can be classified into two types: pseudo-random number generators (PRNG) [1] and true random number generators (TRNG). A PRNG is an algorithm, computational or physical, for generating a sequence of numbers that approximates the properties of random numbers. A physical or hardware version is typically based on stochastic noise or chaotic dynamics in a suitable physical system [2]. Computational PRNGs are based on computational algorithms that generate sequences of numbers of very long periodicity, making them look like true random numbers for sufficiently short sequences. Careful observation over long periods will in principle reveal some kind of pattern or correlation, suggestive of non-randomness.

As far as is known today, the inherent indeterminism or fluctuations in quantum phenomena is the only source of true randomness, an essential ingredient in quantum cryptography [3]. Various proposed underlying physical processes for quantum random number generators (QRNGs) include: quantum measurement of single photons [4, 5], an entangled system [6], coherent states [7, 8] or vacuum states [9]; phase noise [10], spin noise [11], or radioactive decay or photonic emission [12].

In this work, we propose a novel method of QRNG that is a quite different indeterministic paradigm from the above two. It uses *bosonic stimulation* to randomly amplify weak coherent pulses to intense pulses that can be easily detected by a conventional APDs. Bosons (integer-spin quantum particles) obey Bose-Einstein statistics, which entails that the transition probability of a boson into a given final state is enhanced by the presence of identical particles in that state. If there are  $N$  particles in a given quantum state, the probability that an incoming boson makes a transition into that state is proportional to  $N + 1$ . This effect is called bosonic stimulation, and is responsible for coherent matter wave amplification in atomic lasers, as well as the sustenance of a particular mode in a laser cavity, whereby the presence of photons in a particular lasing mode stimulates the emission of more photons into that mode. It provides a new way to combine quantum indeterminism with the tendency of bosons to congregate indistinguishably. We may call this QRNG a *random bosonic stimulator*. Perhaps the practical merit of our proposed QRNG, apart from its harnessing a novel version of quantum indeterminism, is that it simplifies the detection module to the point where it may be accessible to an advanced undergraduate laboratory.

## II. BOSONIC STIMULATION AS A REALIZATION OF THE PÓLYA URN PROBLEM

Consider an urn with  $b$  blue balls and  $r$  red balls. A ball is picked at random and replaced with  $c$  balls of the same color or  $d$  balls of different color. The addition of same color balls results in positive feedback whereas that of

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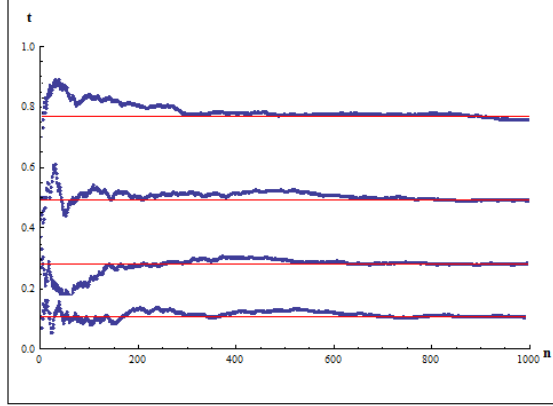


FIG. 1: The relative population of the two ball populations of the Pólya urn, for 4 different runs with the same initial condition  $b(0) = r(0) = 3$ .

different color balls results in negative feedback. During a run of trials, the fractional population of each urn may initially fluctuate randomly, but eventually settles down to a randomly selected *limiting value*  $t$ , giving an instance of symmetry breaking (Figure 1).

Let the initial population of two states labelled as “blue” and “red” be  $b(0)$  and  $r(0)$ , respectively. As the incoming balls start populating the two states, the subsequent growth in population of the modes exhibits Pólya urn behaviour. The probabilistic law of evolution of the fractional population at  $i$ th instance is given by bosonic stimulation to be

$$(b(i), r(i)) \longrightarrow \begin{cases} (b(i) + 1, r(i)) & \text{with probability } \frac{b(i)+1}{b(i)+r(i)+2} \\ (b(i), r(i) + 1) & \text{with probability } \frac{r(i)+1}{b(i)+r(i)+2}. \end{cases} \quad (1)$$

The limiting value  $t \equiv b(i)/(b(i) + r(i))$  for large  $i$  itself varies randomly in the range  $[0, 1]$  from run to run, having a beta distribution

$$f(t; \beta, \rho) = \frac{1}{B(\beta, \rho)} t^{\beta-1} (1-t)^{\rho-1}, \quad (2)$$

where  $B$  is the beta function that normalizes  $f$ ,  $\beta = b'/c$ ,  $\rho = r'/c$ ,  $b' \equiv b(0) + s_b$ ,  $r' \equiv r(0) + s_r$ . For bosonic stimulation, the shifts  $s_b = s_r = 1$ . If the seeding is symmetric, then so is the asymptotic distribution of the limiting values, thereby restoring symmetry, as it should be [13].

Depending on the number of trial runs, the final state can have an arbitrarily large number of bosons. Depending on whether  $t > 0.5$  (blue dominates) or  $t < 0.5$  (red dominates), one generates a random bit 0 or 1. This can serve as the basis of generating random bits at a rate determined by the frequency with which each run can be repeated. Thus, the phenomenon of bosonic stimulation acts as a macroscopic QRNG.

### III. PRACTICAL REALIZATION

A concrete idea for realizing a random bosonic stimulator is to use a lasing medium that supports two radiation modes, for example by vertical and horizontal polarization of the same frequency [14]. A scheme of the proposed experiment is given in Figure 2. Two equal intensity, highly attenuated modes of coherent states are input into a lasing medium. To ensure that the two inputs are synchronized and of equal intensity, a calibrated Mach-Zehnder set-up is used with an attenuated coherent laser pulse fed into one of its input ports. This results in an output consisting of two (unentangled) coherent pulses with half the intensity. A quarter wave plate in one of the arms ensures that the polarization in one arm rotated to be  $90^\circ$  with respect to the other.

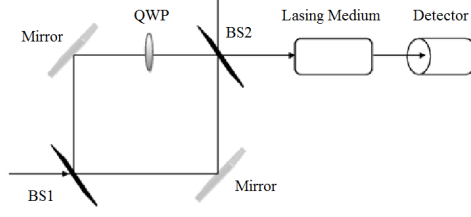


FIG. 2: At the input port, a weak coherent pulse  $|\alpha, H\rangle$  with horizontal polarization and average photon number  $|\alpha|$  enters the experiment. The upper arm branch is rotated to vertical polarization by the quarter-wave plate QWP. The product state  $|\frac{\alpha}{4}, V\rangle|\frac{\alpha}{4}, H\rangle$  enters the lasing medium, where the  $V$  and  $H$  modes participate in bosonic stimulation. In the symmetric case, the detector complex generates a 0 or 1 bit depending on whether  $I_H > I_V$  or the converse, where  $I_H, I_V$  are intensities of the horizontal and vertical polarization output components.

Each mode in a pulse corresponds to a ball color in the Pólya urn problem. Because of bosonic stimulation, the output intensity will randomly favor vertical or horizontal polarization. Let  $I_H$  ( $I_V$ ) denote the intensity of the outcoming light in the horizontal (vertical) polarization mode.

The incoming photon emitted as a result of the coherent de-excitation of the atoms in the medium is assumed to be equally coupled to both modes. Suppose the two modes start in the state  $|n, m\rangle$ , where the first register corresponds to the ‘blue’ mode and the second to the ‘red’ mode. Further let the series of atoms in the population-inverted state be in the initial excited state  $|e, e, \dots\rangle$ . By giving up a photon into the blue or red mode (which could be angular momentum states), the atom is left in the state  $|b\rangle$  or  $|r\rangle$ , assumed to be mutually orthogonal. The joint system of the modes and atoms evolves in a manner analogous to a quantum walk, given by:

$$\begin{aligned}
 |n, m\rangle|e, e, \dots\rangle &\rightarrow \frac{1}{\sqrt{n+m+2}} \left( \sqrt{n+1}|n+1, m\rangle|b\rangle + \sqrt{m+1}|n, m+1\rangle|r\rangle \right) |e \dots\rangle \\
 &\rightarrow \frac{1}{\sqrt{(n+m+2)(n+m+3)}} \left( \sqrt{(n+1)(n+2)}|n+2, m\rangle|b, b\rangle + \sqrt{(n+1)(m+1)}|n+1, m+1\rangle \times \right. \\
 &\quad \left. (|b, r\rangle + |r, b\rangle) + \sqrt{(m+1)(m+2)}|n, m+2\rangle|r, r\rangle \right) | \dots \rangle,
 \end{aligned} \tag{3}$$

and so on. Each term in the superposition is rendered incoherent because it is entangled with a distinct state of atoms, and thus the probability for scattering into a given urn state is quantitatively the same as the classical Pólya urn situation.

Because of the entanglement with the polarization degrees of freedom of the atoms (Eq. (3)), the state of the atoms bears an imprint of the final outcome of laser light. However, as the laser atoms are not individually accessible, the random bit generated is practically unique. From the view point of Monte-Carlo simulations, etc., only the randomness from the laser light read-out will be used. From a cryptographic perspective, the laser system will remain physically well within the encoder unit, preventing its access to a malevolent eavesdropper. Assuming that any possible information leakage through side-channels (like heat radiations from the laser) are reasonably blocked out to the outside world, the assumption of practical uniqueness of the generated randomness applies here, too.

A random bit  $x$  is generated by the detector module, depending on which mode dominates, for example, according to the recipe:

$$\begin{aligned}
 I_H > I_V &\implies x = 0, \\
 I_H < I_V &\implies x = 1.
 \end{aligned} \tag{4}$$

The mean and variance of the distribution  $f(t; \beta, \rho)$  are given, respectively, by

$$\begin{aligned}
 \mu &= \frac{b(0)}{b(0) + r(0)}; \\
 \Delta^2 &= \frac{b(0)r(0)}{(b(0) + r(0))^2(b(0) + r(0) + 1)}.
 \end{aligned} \tag{5}$$

The value  $t$  obtained will tend to peak towards the mean, with ever lower variance if one or both of the initial populations are large. If the instrument function of the detector is denoted by a normal distribution  $e$  with FWHM  $s$ , the observed distribution is the convolution  $p(x) = \int f(t; \beta, \rho) e(x - t) dt$ . It is important that we use sufficiently weak pulses obtained by attenuating coherent light sources, so that  $s \ll \Delta$ . This ensures that the quantum randomness dominates over stochastic noise in the detector's reading. The distribution  $f(t; \beta = 1, \rho = 1)$  is uniform over  $[0, 1]$ , and thus the performance of the random bosonic stimulator is least affected by detector tolerance. This means that the two inputs should ideally be vacuum modes.

A challenge will be to ensure that the beam splitters are truly 50-50 and the coupling of the excited atoms is equal to the two modes. However, even if this is not so, Eq. (2) and the prescription (4) can be suitably generalized to yield a random bit of uniform distribution. Suppose a photon couples to the  $H$  mode stronger than to the  $V$  mode (because of an atomic or beam-splitter feature) by a factor  $(1 + \epsilon)$ , then the new distribution can be shown to be Eq. (2), but with  $\beta \rightarrow \beta' = \beta(1 + \epsilon)$ . Let  $t_{1/2}$  be the median of the distribution  $f$ , defined such that  $\int_{t=0}^{t_{1/2}} f(t; \beta', \rho) = \frac{1}{2}$ , where  $f$  is the beta distribution (2). Then, we obtain our uniformly random bit by replacing prescription (4) by:

$$\begin{aligned} t < t_{1/2} &\implies x = 0 \\ t \geq t_{1/2} &\implies x = 1 \end{aligned} \quad (6)$$

To be precise, the above numbers assume that the input modes are pure number states.

More realistically, taking into account that they are coherent states, we must replace Eq. (2) by

$$f'(t; \lambda) = \sum_{\beta, \rho} \frac{1}{B(\beta, \rho)} t^{\beta-1} (1-t)^{\rho-1} P(\lambda, \beta) P(\lambda, \rho), \quad (7)$$

where  $P(\lambda, x)$  is the Poisson distribution of  $x$  with mean  $\lambda$ . Furthermore, in practice we may have to let  $\lambda$  to range over an interval because of practical difficulties of producing the same exact degree of attenuation on each run. It can be shown that this added complication does not affect our main results.

For sufficiently low noise in each run, two bits may also be generated per run according to the recipe:

$$\begin{aligned} t < t_{1/4} &\implies x = 00 \\ t_{1/4} \leq t < t_{1/2} &\implies x = 01 \\ t_{1/2} \leq t < t_{3/4} &\implies x = 10 \\ t \geq t_{3/4} &\implies x = 11, \end{aligned} \quad (8)$$

where  $t_\xi$  is defined such that  $\int_0^{t_\xi} f(t; \beta', \rho) = \xi$ . More generally, to generate  $n$  bits, the noise level should be lower than  $2^{-n}$ .

#### IV. DISCUSSION AND CONCLUSIONS

We have proposed a novel QRNG principle, based on bosonic stimulation, in which, while the state preparation procedure presents experimental challenges, the detection and read-out parts are easier to implement. In an actual experiment, it is possible that systematic experimental biases might introduce correlations into the sequence of bits produced, thereby degrading the randomness. Statistical analyses like the Diehard tests and National Institute of Standards and Testing (NIST) suite of tests for randomness have to be carried out to know the quality of randomness and improve upon it.

Some implementational details are worth noting. Since the required light is not for communication to a geographically distant station, any laser system that can be conveniently tuned to operator at 800 nm, where good detectors (APDs) are available, is suitable for our purpose. A good candidate is femtosecond Titanium-Sapphire lasers, for which pulse repetition rates upto a few gigahertz can be obtained.

Further, the potential problem posed by the cavity's unequal coupling to the two modes can be taken care of by "biasing" the comparator (as clarified above). Care must be taken to choosing the operational/differential amplifier (op amp) IC designed to work as comparator, so that their frequency specifications are appropriate for the pulse rate of the laser. Even with the low resolution provided by two-bit comparators, for a Titanium-Sapphire laser, mentioned above, this yields a random bit-rate of the order of  $10^9$  bits per second, which is comparable to bit-rates in state-of-the-art QRNGs.

As another possible realization of bosonic stimulation based QRNG, one may consider photonic band gap materials, results pertaining to which are presented elsewhere.

The present method of generating randomness is based on the essentially quantum feature of being able to turn a distinct bosonic particle into an indistinguishable part of a collective object, which is the mode or a condensate. The notion of identity in classical logic and philosophy is as such incapable of handling this situation, requiring recourse to concepts like quasi-set theory [16]. We think that, as in the case of the application of quantum nonlocality to cryptography [17], our method of generating randomness based on bosonic indistinguishability, can perhaps be termed as another instance of *applied philosophy*!

Here we have presumed the axiomatic indeterminism of quantum mechanics. This assumption would be falsified if a (nonlocal) hidden variable theory were able to explain quantum mechanics in the future. As this seems unlikely, a QRNG still seems the best bet for a source of genuine and unique randomness.

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